October 4th, 2022

Modeling the Relationship Between Paper Helicopter Wing Length and Drop Time

Helen Moses, Ben Griesel, Elena Ea

Abstract

This study analyzes the relationship between paper helicopter wing length and drop time. We compared the mean drop time between four different groups of wing length n = 20 (5.5 inches, 6.5 inches, 7.5 inches, 8.5 inches). An ANOVA analysis of the data yielded strong evidence in favor of an 8.5 inches wing length being the optimum wing length to obtain the longest flight time.

Introduction

In this study we will determine whether the wing length affects the time it takes for a paper helicopter to fall to the ground. Helicopters are unique vehicles which are kept in the air almost completely due to their wing design. This characteristic differs from airplanes, jets, and other aircrafts which utilize engine and propulsion systems which provide the majority of thrust forward. Because of the importance wings hold for the flight of a helicopter, we are modeling our experiment attempting to find the optimal wing length needed to maximize the flight time of a cardstock paper helicopter. If the wing length did not affect the flight times of the helicopters, then there would be no effect of a helicopter being in one group versus another in our regression model and no difference in the mean flight times of the wing length groups in our one-way ANOVA model. Thus, our null hypothesis are H_0: $\boldsymbol{\beta}_1 = 0$ (the slope of the difference in mean drop times is 0), and H_0: $\mu_1 = \mu_2 = \mu_3 = \mu_4$ (none of the wing length group flight time means are different for each other) for the regression and one way ANOVA models respectively. Our alternative hypothesis are H_a: $\boldsymbol{\beta}_1$ =! 0 (the slope of the difference in mean flight times is not zero). and H_a: μ_i =! μ_j (at least one of the group mean flight times is different).

Methods

To create our paper airplanes, we used cardstock paper, a ruler, and scissors. We drew 4 different models of the helicopter by copying the given model and measurements on the Chapter 2 Research Project page (63), the only difference being that we cut different wing lengths for each of the 4 models: 5.5 inches, 6.5 inches, 7.5 inches, and 8.5 inches. We then cut out 5 helicopters for each of the models and labeled each with their wing length, so we had 20 helicopters in total, and folded the wings so they faced in opposite directions. A paper clip was attached to the base of the helicopter to give enough weight for the helicopters to properly spin. We dropped each helicopter from the same height (distance from the second floor of CMC to the first floor) with the paper clip side pointing down to the ground. We recorded the time from when the helicopter was released until it hit the ground. We chose a random airplane each time from a pile so the wing lengths chosen were random. To control for variation in timing, the same person randomly picked up a helicopter and dropped it precisely the same way for the entire process and another person was responsible for recording the time for the entire process. We recorded our times in seconds. Because we were only interested in the relationship between wing length and drop time, we had to make sure we kept all the other extraneous variables constant. We made sure to drop all of our helicopters within a fifteen-minute time period and all in the exact same location inside to ensure the humidity and wind speeds were the same. The inside location was preferable because the quality of the air (humidity and wind speed) is much more consistent than in an outside location.

Results

The data collected appears to have two severe outliers (*Figures 1 and 2*). However, the general trend of the data suggests that as the helicopter wing size increases, so does the drop time (*Figures 1 and 2*). Conversely, the initial ANOVA test on the data provided weak evidence against there being a difference in the mean drop times between any of the groups (f-value: 1.999, p-value: .155, df: 3). Further analysis on the data revealed that not all of the model assumptions were satisfied. Most importantly, the plot of the errors is not normally distributed. In addition, there is not equal variance among the residuals, and the largest standard deviation is more than two times greater than the smallest standard deviation. No common transformations allowed the data to meet the model assumptions. Instead, the two obvious outliers were removed from the data set to allow for an analysis on their influence.

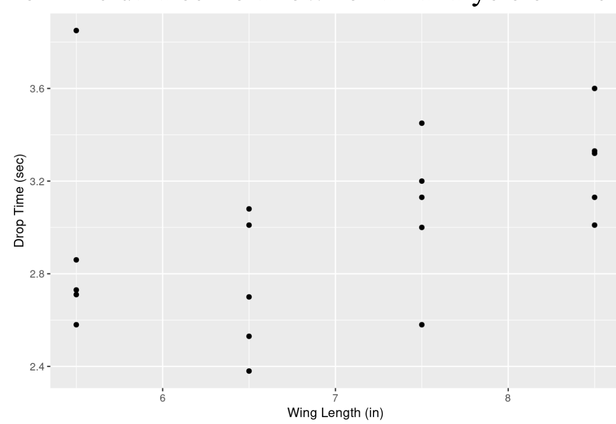

Figure 1. Scatterplot of the original data

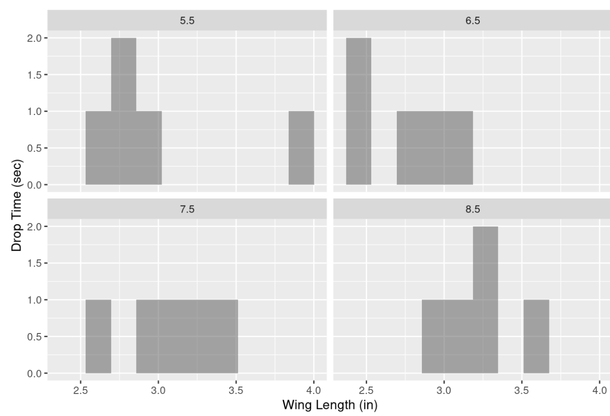

Figure 2. Histogram of the original data

Without the outliers, the data appears less varied (*Figures 2 and 4)*. In addition, further analysis concluded that all of the model assumptions are now satisfied. However, it should be noted that the largest standard deviation is still slightly more than two times the smallest standard deviation.

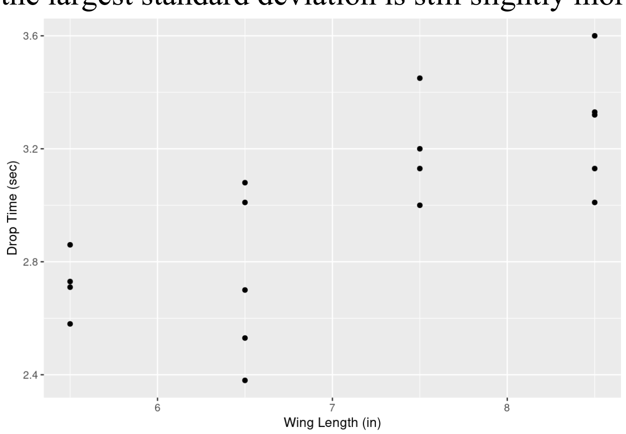

Figure 3. Scatterplot of data with outliers excluded

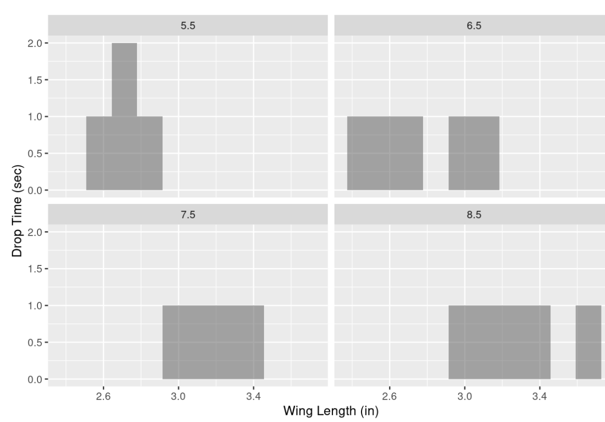

Figure 4. Histogram of data with outliers excluded

A new ANOVA analysis revealed that there is statistically discernable evidence to reject the null hypothesis and assert at least one of the wing lengths varies in mean drop time from another (f-value: 7.774, p-value: 0.003, df: 3). A post-hoc analysis (*Table 1*) concluded the optimal wing length, from the wing lengths tested, is 8.5 inches, as we are 95% confident the mean drop time is between .08 and 1.04 seconds longer than the 5.5 inches wing length drop time. Because the ANOVA test provided evidence for a linear relationship, we decided to fit a linear regression model to see if that model represented the data better than the ANOVA model (*Figure 5*). The model assumptions were satisfied to the same degree as with the ANOVA model assumptions. The regression model provides strong evidence in favor of one of the mean drop times being statistically significant from another (t-value: 4.376, p-value: <.001).

Comparison	Confidence Interval
5.5(in) and 6.5(in)	(-0.50, 0.46)
5.5(in) and 7.5(in)	(-0.98, 0.03)
5.5(in) and 8.5(in)	(-1.04, -0.08)
6.5(in) and 7.5(in)	(-0.94, 0.03)
6.5(in) and 8.5(in)	(-1.00, -0.08)
7.5(in) and 8.5(in)	(-0.56, 0.40)

Table 1. Post-hoc analysis on ANOVA data

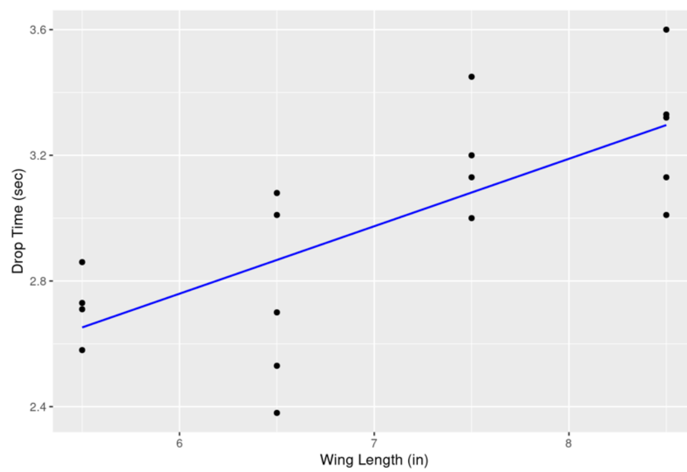

Figure 5. Linear regression model on data excluding the outliers ($\hat{y}_{DropTime} = 1.471 + 0.049x_{WingLength}$)

Discussion

Both models suggest there is a statistically discernible difference between mean drop times, and that the preferable wing length is 8.5 inches. However, the linear regression model suggest that the evidence is slightly stronger than the ANOVA model does. We believe this is due to the linear regression model not fitting the data quite as well. We can see that the regression line only goes through the middle of the 5.5 inches group and the 8.5 inches group (*Figure 5.)* This highlights that the data, while trending upwards, might not be increasing by the same amount each incremental step. In other words, this data cannot be described with one slope. In addition, the regression model could lead to the false assumption that the relationship will stay linear forever, and that no matter what, a longer wing length will lead to a longer flight time. For this reason, we find the evidence produced by the ANOVA model to be more informative. It is important to note that while our data produced evidence in favor of the 8.5 inches wing length being optimal for the longest drop time, this does not indicate it is optimal over lengths we did not test. For this reason, a potential area for future study would be to extend this experiment to much longer wing lengths. By looking at longer wing lengths, we could explore a potential relationship in which a longer wing length might add to a quicker drop time because the added weight will outweigh the benefit of the longer wing.